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Symmetries, large leptonic mixing and a fourth generation

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Abstract: We show that large leptonic mixing occurs most naturally in the framework of the Sandard Model just by adding a fourth generation. One can then construct a small Z_4 discrete symmetry, instead of the large $S_{4L} \times S_{4R}$, which requires that the neutrino as well as the charged lepton mass matrices be proportional to a 4×4 democratic mass matrix, where all entries are equal to unity. Without considering the see-saw mechanism, or other more elaborate extensions of the SM, and contrary to the case with only 3 generations, large leptonic mixing is obtained when the symmetry is broken.

Keywords: Standard Model, Neutrino Physics.

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Contents

1. Introduction

Recently [1], it was suggested that the existence a fourth generation of fermions is in agreement with the latest electroweak precision data. However, this simple extension of the Standard Model (SM) could also be a natural way to explain the smallness of the masses of the first 3 generations of neutrinos. It was shown [2], for the lepton sector and within a democratic weak basis, that small masses for the first 3 neutrinos are compatible with large mixing angles, assuming that neutrinos are of the Dirac type. In the democratic 4×4 limit, all Yukawa couplings between the different generations of leptons are equal, and only the fourth generation acquires mass, for each lepton sector. Democracy provides an alternative explanation (e.g. to the see-saw mechanism) for the smallness of the neutrino masses as it does not require very small Yukawa couplings for the neutrinos but only that they be almost equal, and this may result from a discrete (permutation) symmetry and its breaking.

In addition, data on neutrinos have provided clear evidence pointing towards neutrino oscillations with very large lepton mixing between the 3 known neutrino families [3]. However, unlike in the quark sector, where simple S_3 permutation symmetries can generate the general features of quark masses and mixings, it was found [4] that it is impossible to obtain large leptonic mixing angles with any general symmetry (or its breaking) of only the 3 known generations, without having to consider the see-saw mechanism or a more elaborate extension of the SM. Therefore, if such symmetries exist, they must be realized in more extended scenarios.

In this Letter, we will show that it is exactly within this simple extension of the SM (unless, as explained, one considers the see-saw mechanism or other more elaborate extensions of the SM), just by adding a fourth generation of SM lepton doublets and singlets, that one may obtain large mixing angles for the leptons after breaking the symmetry.

JHEP12(2002)036

2. Large leptonic mixing and symmetry

First, we consider the large leptonic mixing consistent with the present data on neutrinos. One of the most attractive scenarios for the mixing is an orthogonal matrix of type

$$F^T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \tag{2.1}$$

from which we find large mixing angles, $\sin^2(2\theta_{\rm atm}) = 8/9$ for the atmospheric neutrinos, and $\sin^2(2\theta_{\rm sol}) = 1$ for the solar neutrinos. One may think of different frameworks, where a leptonic mixing matrix approximate to F^T arises from a simple and obvious structure. Of course, the simplest scenario, which reproduces F^T, can be obtained if one takes different structures for the charged lepton and neutrino mass matrices. In the limit where the charged lepton mass matrix is proportional to the democratic mass matrix Δ, where all entries are equal to 1, while the neutrino mass matrix is proportional to $1\!\!I$, the lepton mixing matrix will be the transpose of the orthogonal matrix which diagonalizes Δ, and this is just F^T [5].

Another approach was considered in ref. [6]. In the framework of the universal strength for Yukawa couplings (USY), it was proven that large lepton mixing is compatible with all lepton mass matrices having the same structure, of the form

$$M = c \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\beta} & 1 \\ 1 & 1 & e^{i\gamma} \end{bmatrix}. \tag{2.2}$$

The hierarchy of the charged leptons requires $\alpha_e, \beta_e, \gamma_e$ to be small and M_e is near to the democratic limit Δ. For the neutrino mass matrix, one must take $\alpha_\nu, \beta_\nu, \gamma_\nu$ to be large, otherwise the leptonic mixing angles would not be large either. Indeed, if one takes $\alpha_\nu, \beta_\nu, \gamma_\nu$ to be near to $2\pi/3$, a lepton mixing matrix near to F^T can be obtained. In the limit $\alpha_\nu = \beta_\nu = \gamma_\nu = 2\pi/3$, the neutrino mass matrix will be proportional to a unitary matrix W, and the neutrinos will be degenerate. The mixing F^T will result from a small perturbation of W, just by taking $\alpha_\nu = \beta_\nu$ and γ_ν slightly different from $2\pi/3$.

However, as pointed out in ref. [4], both structures, i.e. $M_e = \Delta$ while $M_\nu = 1\!\!I$ or the pattern of eq. (2.2), could never be the result of a simple symmetry or of its breaking. More precisely, there exists no symmetry, transforming the lepton fields

$$\begin{array}{ccc} L_i & \to & P_{ij}\ L_j \\ e_{iR} & \to & Q_{ij}\ e_{jR} \end{array} \quad \text{or} \quad \begin{array}{ccc} L_i & \to & P_{ij}\ L_j \\ e_{iR} & \to & Q_{ij}\ e_{jR} \\ \nu_{iR} & \to & R_{ij}\ \nu_{jR} \end{array} \tag{2.3}$$

(depending on whether neutrinos are Majorana or Dirac[1]) and imposing invariance conditions for the lepton mass matrices

$$P^\dagger \cdot M_e \cdot Q = M_e\,, \qquad P^T \cdot M_\nu \cdot P = M_\nu \qquad \text{or} \qquad P^\dagger \cdot M_\nu \cdot R = M_\nu\,, \tag{2.4}$$

[1]We do not consider (heavy) mass terms for the right-handed neutrinos and the see-saw mechanism. In fact, in the latter context, the statement we make here is not necessarily true [8].

that would force the charged lepton mass matrix to be proportional to the democratic limit Δ, while, at the same time, requiring that the neutrino mass matrix be given by a matrix proportional to $\mathbb{I}$ or by eq. (2.2) with $\alpha_\nu = \beta_\nu = \gamma_\nu = 2\pi/3$. On the contrary, it was proven that, if such a symmetry exists, the neutrino and the charged lepton mass matrices have to be related as

$$M_\nu^\dagger M_\nu \Delta = p\Delta \qquad \text{or} \quad M_\nu M_\nu^\dagger \Delta = p\Delta\,, \tag{2.5}$$

and this relation[2] implies small mixing angles, insufficient to solve the atmospheric neutrino problem even after the symmetry is broken.

The question then arises of whether it is possible to have a framework in which the breaking of flavour symmetries will generate large leptonic mixing near to F^T, and how. Before studying the case of 4 generations, we give two examples in which this was achieved in the context of some elaborate framework:

- Huge flavour symmetries [7]: a huge flavour symmetry $\mathrm{O}(3)_L \times \mathrm{O}(3)_R$ can, of course, force the (Majorana) neutrino mass matrix to be proportional to $\mathbb{I}$ however, the (usual) charged lepton mass matrix will be zero. With extra Higgs fields and appropriate mass scales it is then possible to construct a large extra term in the lagrangian that will result in a charged lepton mass matrix proportional to the democratic limit Δ. As explained, the mixing will be just F^T.

- See-saw mechanism: another important example, which generates large leptonic mixing near to F^T, was given in ref. [8]. It was proven that, within the see-saw mechanism, a Z_3 symmetry can force all lepton mass matrices to be proportional to Δ, but does not require small mixing. It is crucial to notice that, in the see-saw mechanism, there is an additional matrix that respects eq. (2.4): the heavy neutrino Majorana mass matrix. It is this extra ingredient that prevents small mixing. The heavy Majorana neutrino mass matrix, which is proportional to Δ, has no (or a singular) inverse. Therefore, the effective neutrino mass matrix can only be computed if a suitable perturbation is added. It is then the combined perturbations of the different mass matrices that will make it possible to a have large leptonic mixing matrix near to F^T.

Still, might it not be possible to obtain the same result, i.e. some framework in which the breaking of flavour symmetries will generate large leptonic mixing near to F^T, in a much simpler context? We will show that this is indeed the case. Large leptonic mixing near to F^T occurs most naturally in the framework of the SM with just one simple extension and in connection with a discrete symmetry. Adding a fourth generation to the SM (making sure that the masses and mixings of the heavy extra particles respect the experimental constraints), one can construct a simple discrete symmetry, which will require that the neutrino as well as the charged lepton mass matrices be proportional to a 4×4 democratic

[2]In addition, it was shown that the symmetry cannot prevent the neutrino mass matrix from having a part which is proportional to Δ.

mass matrix Δ, while at the same time generating large leptonic mixings through the breaking. Thus, in this case, large leptonic mixing is consistent with a (discrete) symmetry and its breaking. In this sence (it is also a most simple extension of the SM) we call it "natural"; we do not violate 't Hooft's naturalness principle [9] as in the case with 3 generations, where it was impossible, outside the see-saw mechanism or other more elaborate extensions of the SM [4], to obtain large mixing in connection with a symmetry. Furthermore, we will show that it suffices to consider a small Z_4 discrete symmetry instead of the large $S_{4L} \times S_{4R}$.

3. A fourth generation and a Z_4 symmetry

Consider the SM with Dirac neutrinos,[3] and one extra generation of lepton doublets and lepton singlets:

$$-L = \lambda^e_{ij}\, \overline{L}_i \phi e_{jR} + \lambda^\nu_{ij} \overline{L}_i \tilde{\phi} \nu_{jR} + \text{h.c.}\,; \qquad i,j = 1,2,3,4\,. \tag{3.1}$$

In order to obtain mass matrices for the charged leptons and neutrinos proportional to a 4×4 democratic Δ, we impose a symmetry on the lepton fields, which is realized in following way:

$$\begin{array}{l} L_i \to P^\dagger_{ij}\, L_j \\ e_{iR} \to P_{ij}\, e_{jR}\,; \\ \nu_{iR} \to P_{ij}\, \nu_{jR} \end{array} \qquad P = \mathbb{1} - \frac{1+i}{4}\,\Delta\,. \tag{3.2}$$

It is easy to check that this is indeed a Z_4 discrete symmetry and that the charged lepton and neutrino mass matrices must then be proportional to Δ. Using $\Delta^2 = 4\Delta$, one can verify that P is unitary, $P^2 = \mathbb{1} - \Delta/2$, $P^3 = P^\dagger$ and $P^4 = \mathbb{1}$. Both the neutrino and charged lepton mass matrix respect the relation

$$P \cdot M \cdot P = M\,. \tag{3.3}$$

From this equation it follows that

$$M = \frac{1}{3}\left(P \cdot M \cdot P + P^2 \cdot M \cdot P^2 + P^3 \cdot M \cdot P^3\right). \tag{3.4}$$

Inserting the expressions for P, $P^2$and P^3, one finds that any mass matrix that obeys eq. (3.3), and subsequently eq. (3.4), must fulfil the constraint

$$M = \frac{1}{4}\left(\Delta \cdot M \cdot \Delta\right). \tag{3.5}$$

Finally, using the property $\Delta M \Delta = m\ \Delta$, where $m = \sum M_{ij}$ (valid for all matrices) one obtains that M has to be proportional to Δ.

[3] Again, we do not allow for a (heavy) Majorana mass term for the right-handed neutrinos. These can be avoided, e.g. by an extra discrete symmetry where $\nu_{jR} \to i\nu_{jR}$, $j = 1,2,3,4$. In a more complete scenario, one must also consider an extra generation of quark doublets and up and down singlets, e.g. in order to have anomaly cancelation.

JHEP12(2002)036

At this stage, it should be mentioned that the (usual) SM charged leptons and neutrinos of the first 3 generations all have zero mass and that the lepton mixing for the 3×3 sector is (as yet) completely arbitrary. The unitary matrix which diagonalizes the 4×4 democratic limit Δ is not uniquely defined and can be written as $F_4\cdot U$, where U is an arbitrary unitary matrix that has significant elements only in the 3×3 sector (i.e. $U_{i4}=0$, $i=1,2,3$) and

$$F_4=\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}. \tag{3.6}$$

Thus, it is the breaking of the Z_4 discrete symmetry that will give masses to the 3 first generations and determine the lepton mixing.

By adding a small term to the democratic limit Δ, we break the Z_4 symmetry. Obviously, it is possible to have different breaking patterns and leptonic mixing. As an example, just to illustrate the possibility of having large leptonic mixing near to F^T we choose for the charged lepton and neutrino mass matrices the following simple pattern:

$$M_{e,\nu}=k_{e,\nu}\;(\Delta+P_{e,\nu})\,, \tag{3.7}$$

where

$$P_e=\mathrm{diag}(0,a_e,b_e,c_e);\qquad P_\nu=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a_\nu & 0 & a_\nu \\ 0 & 0 & b_\nu & b_\nu \\ 0 & a_\nu & b_\nu & b_\nu+a_\nu \end{bmatrix}.$$

For simplicity, we take real parameters. There is not yet any (or no) experimental evidence for CP violation in the lepton sector. The hierarchy of the charged leptons requires that $a_e\ll b_e\ll c_e\ll 1$. From the neutrino hierarchy $\Delta m^2_{21}\ll\Delta m^2_{32}$ we obtain also $a_\nu\ll b_\nu\ll 1$. In addition, we get the relations $m^2_{\nu_2}\approx\Delta m^2_{21}$and $m^2_{\nu_3}\approx\Delta m^2_{32}$ because of the restricted number of parameters for the neutrino mass matrix (and $a_\nu\ll b_\nu$). From the invariants, $\mathrm{tr}(M)$, $\chi_2(M)$, (M) for the charged lepton and neutrino mass matrix, it is easy to derive the following first order approximations:

$$\begin{aligned} k_e &= \frac{1}{4}m_{\ell_4}\,, & a_e &= \frac{8m_e}{m_{\ell_4}}\,, & |b_e| &= \frac{6m_\mu}{m_{\ell_4}}\,, & c_e &= \frac{16m_\tau}{3m_{\ell_4}}\,, \\ k_\nu &= \frac{1}{4}m_4\,, & a_\nu &= \frac{4m_2}{m_4}\,, & b_\nu &= \frac{4m_3}{m_4}\,, \end{aligned} \tag{3.8}$$

where m_{ℓ_4} and m_4 are the masses of the (extra) fourth charged lepton and neutrino. At tree level, one neutrino is massless, $m_1=0$. In a first order approximation, we find $(a_\nu/b_\nu)^2=\Delta m^2_{21}/\Delta m^2_{32}$ and $m_{\nu_2}=(\Delta m^2_{21})^{1/2}$, $m_{\nu_3}=(\Delta m^2_{32})^{1/2}$.

The hierarchy of the parameters of the neutrino and charged lepton mass matrices allows for a straightforward first order computation of the orthogonal matrices that diagonalize M_e and M_ν. After a weak basis transformation to a heavy basis, where the Δ part in eq. (3.7) becomes diagonal, i.e. $M_{e,\nu}\to F_4^T\cdot M_{e,\nu}\cdot F_4$, we need only find orthogonal matrices that will eliminate, from the first two horizontal lines of M_e and M_ν, the parameter that will be next in the order. So, from the first horizontal line of M_e we eliminate b_e and c_e, and from the second line we eliminate c_e. For the neutrino mass matrix, as the first eigenvalue is zero, we eliminate only b_ν from the second line. Thus, we obtain in approximation for the matrices that diagonalize M_e and M_ν of our ansatz in eq. (3.7):

$$U_e = F_4 \cdot F_3 \,, \qquad U_\nu = F_4 \,, \tag{3.9}$$

where F_3 is the 4×4 orthogonal matrix that contains, in the 3×3 SM sector, exactly our original F (its transpose was given in eq. (2.1)), and $(F_3)_{i4} = 0$, $i = 1, 2, 3$. As a final step, to obtain the total orthogonal matrices that diagonalize M_e and M_ν we have to multiply U_e and U_ν by matrices I_e and I_ν which are very near to $\mathbb{I}$. For the charged leptons $(I_e)_{ij} = O(m_{\ell_i}/m_{\ell_j})$ for $i < j$, and for the neutrinos the angles are almost insignificant. Therefore, in a first order approximation, the lepton mixing matrix will be

$$V = U_e^T \cdot U_\nu = F_3^T \,,$$

which coincides exactly with our original F^T for the first 3 generations and the 3×3 SM sector. The mixing with the fourth generation is at most $O(m_\tau/m_{\ell_4})$ between the 3rd and 4th families. We give a numerical example.

Input:

$$\begin{aligned} a_e &= 4.11 \times 10^{-5} & b_e &= -6.34 \times 10^{-3} & c_e &= 9.66 \times 10^{-2} \\ a_\nu &= 6.0 \times 10^{-13} & b_\nu &= 5.1 \times 10^{-12} \,. \end{aligned} \tag{3.10}$$

For the heavy extra charged lepton and neutrino, we have chosen masses [4] $m_{\ell_4} = 100\,\mathrm{GeV}$ and $m_4 = 50\,\mathrm{GeV}$.

Output:

$$\begin{aligned} m_e &= 0.511\,\mathrm{MeV}\,, & m_\mu &= 105.7\,\mathrm{MeV}\,, & m_\tau &= 1777\,\mathrm{MeV} \\ \Delta m_{21}^2 &= 5.6 \times 10^{-5}\,\mathrm{eV}^2\,, & \Delta m_{32}^2 &= 4.0 \times 10^{-3}\,\mathrm{eV}^2\,, \\ m_{\nu_3} &= 0.064\,\mathrm{eV} \end{aligned} \tag{3.11}$$

and

$$|V| = \begin{bmatrix} 0.706 & 0.708 & 2.14 \times 10^{-3} & 7.3 \times 10^{-6} \\ 0.422 & 0.418 & 0.805 & 8.6 \times 10^{-4} \\ 0.569 & 0.569 & 0.594 & 1.1 \times 10^{-2} \\ 6.5 \times 10^{-3} & 6.5 \times 10^{-3} & 5.7 \times 10^{-3} & 0.9999 \end{bmatrix} \tag{3.12}$$

from which we obtain

$$\sin^2(2\theta_{\mathrm{atm}}) = 0.913, \quad \sin^2(2\theta_{\mathrm{sol}}) = 1.0 \,. \tag{3.13}$$

The value for $|V_{\tau 4}|^2 = 1.17 \times 10^{-4}$ is above the value permitted by the data available from DELPHI [10] where $|V_{\tau 4}|^2 < 3 \times 10^{-5}$ for $m_{\nu_4} = 50\,\mathrm{GeV}$. The values for the atmospheric and solar neutrino mixing are in good agreement with the values for the Large Mixing

[4] These masses are also in accord with the values given by Novikov, Okun, Rozanov and Vysotsky [1]

JHEP12(2002)036

Angle (LMA) scenario from previous analysis. At present, slightly different values, where $\sin^2(2\theta_{\rm atm})$ is almost maximal, near to 1.0, and $\sin^2(2\theta_{\rm sol}$ is somewhat smaller than 1, seem to be favoured. However, it is not yet completely clear what the parameter range is for the neutrino masses and mixings. In any case, both scenarios can be accommodated in the scheme presented here.

4. Conclusions

We have shown that large leptonic mixing occurs most naturally in the framework of the SM, just by adding a fourth generation to the SM. One can then construct a very simple discrete symmetry that will require the neutrino as well as the charged lepton mass matrices be proportional to a 4×4 democratic mass matrix Δ. Without considering the see-saw mechanism, or other more elaborate extensions of the SM, and contrary to the case with only 3 generations, large leptonic mixing is obtained when the symmetry is broken. Furthermore, it was shown that it suffices to consider a small Z_4 discrete symmetry instead of the large $S_{4L} \times S_{4R}$.

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References

[1] V.A. Novikov, L.B. Okun, A.N. Rozanov and M.I. Vysotsky, *Mass of the Higgs versus fourth generation masses*, hep-ph/0203132; *Extra generations and discrepancies of electroweak precision data*, *Phys. Lett.* **B 529** (2002) 111 [hep-ph/0111028];
E. Arik, O. Çakir, S.A. Çetin and S. Sultansoy, *Consequences of the extra SM families on the Higgs boson production at Tevatron and LHC*, *Phys. Rev.* **D 66** (2002) 033003 [hep-ph/0203257].

[2] J.I. Silva-Marcos, *Alternative to the seesaw mechanism*, *Phys. Rev.* **D 59** (1999) 091301; it was brought to my attention that, independently, in 1992, H. Fritzsch also formulated this idea in an unpublished paper, MPI-Ph/92-42. For other models with a fourth generation of leptons see also R.N. Mohapatra and S. Nussinov, *Bimaximal neutrino mixing and neutrino mass matrix*, *Phys. Rev.* **D 60** (1999) 013002 [hep-ph/9809415].

[3] Super-Kamiokande collaboration, T. Toshito, *Super-kamiokande atmospheric neutrino results*, hep-ex/0105023;
Super-Kamiokande collaboration, S. Fukuda et al., *Solar* ^{8}B *and hep neutrino measurements from 1258 days of super-Kamiokande data*, *Phys. Rev. Lett.* **86** (2001) 5651 [hep-ex/0103032]; *Constraints on neutrino oscillations using 1258 days of super-Kamiokande solar neutrino data*, *Phys. Rev. Lett.* **86** (2001) 5656 [hep-ex/0103033];
MACRO collaboration, M. Ambrosio et al., *Matter effects in upward-going muons and sterile neutrino oscillations*, *Phys. Lett.* **B 517** (2001) 59 [hep-ex/0106049];
GALLEX collaboration, W. Hampel et al., *Gallex solar neutrino observations: results for Gallex IV*, *Phys. Lett.* **B 447** (1999) 127;

JHEP12(2002)036

SAGE collaboration, J.N. Abdurashitov et al., *Measurement of the solar neutrino capture rate with gallium metal*, *Phys. Rev.* **D 60** (1999) 055801 [astro-ph/9907113];
GNO collaboration, M. Altmann et al., *GNO solar neutrino observations: results for GNO I*, *Phys. Lett.* **B 490** (2000) 16 [hep-ex/0006034];
SNO collaboration, *Measurement of the rate of* $\nu_e + d \to p + p + e^-$ *interactions produced by* ^8B *solar neutrinos at the Sudbury Neutrino Observatory*, *Phys. Rev. Lett.* **87** (2001) 71301.

[4] J.I. Silva-Marcos, *The problem of large leptonic mixing*, hep-ph/0204051.

[5] H. Fritzsch and Z.-z. Xing, *Large leptonic flavor mixing and the mass spectrum of leptons*, *Phys. Lett.* **B 440** (1998) 313 [hep-ph/9808272];
M. Fukugita, M. Tanimoto and T. Yanagida, *Atmospheric neutrino oscillation and a phenomenological lepton mass matrix*, *Phys. Rev.* **D 57** (1998) 4429 [hep-ph/9709388].

[6] G.C. Branco, M.N. Rebelo and J.I. Silva-Marcos, *Large neutrino mixing with universal strength of Yukawa couplings*, *Phys. Rev.* **D 62** (2000) 073004 [hep-ph/9906368]; *Degenerate and quasi degenerate majorana neutrinos*, *Phys. Rev. Lett.* **82** (1999) 683 [hep-ph/9810328].

[7] M. Tanimoto, T. Watari and T. Yanagida, *Democratic mass matrices from broken* $O(3)_L \times O(3)_R$ *flavor symmetry*, *Phys. Lett.* **B 461** (1999) 345 [hep-ph/9904338].

[8] G.C. Branco and J.I. Silva-Marcos, *The symmetry behind extended flavour democracy and large leptonic mixing*, *Phys. Lett.* **B 526** (2002) 104 [hep-ph/0106125].

[9] G. 't Hooft, *Naturalness, chiral symmetry and spontaneous chiral symmetry breaking*, lecture given at Cargèse Summer Institute 1979, p. 135.

[10] Particle Data Group, *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.